SE Lie Theory Workshop 2014: Contributed Talks

Speaker: Irfan Bagci
Affiliation: University of North Georgia

Title: *Integral forms and integral bases for universal enveloping algebras of map superalgebras*

Abstract: Let $\mathfrak{g}$ be a finite dimensional complex simple Lie superalgebra and A be a commutative, associative algebra with unity over the $\mathbb{C}$. In this talk we will define an integral form for the universal enveloping algebra of the map superalgebra $\mathfrak{g} \otimes A$ and exhibit an explicit integral basis for this integral form.

Speaker: Huanchen Bao
Affiliation: University of Virginia

Title: *Geometric Schur Duality*

Abstract: Recently a theory of canonical bases arising from coideal subalgebras of quantum groups has been developed by the speaker and Weiqiang Wang, which has been used in formulating a new approach to Kazhdan-Lusztig theory of classical type. (This will be addressed in Wang's talks.) In this talk, we give a geometric realization of those (modified) coideal subalgerbas using type B/C flag varieties. As a consequence of the geometric construction, we show the existence of canonical bases on the (modified) coideal subalgebras. This is joint work with Jonathan Kujawa, Yiqiang Li, and Weiqiang Wang.

Speaker: Nathaniel Bushek
Affiliation: University of North Carolina

Title: *Hitchin's Conjecture for Simply-laced Lie Algebras implies that for any Simple Lie Algebra*

Abstract: Let $\mathfrak{g}$ be any simple Lie algebra over $\mathbb{C}$. Recall there exists a princpal TDS embedding of $\mathfrak{sl}_2$ into $\mathfrak{g}$ passing through a principal nilpotent of $\mathfrak{g}$. $\wedge(\mathfrak{g}^*)^{\mathfrak{g}}$ is generated (in the super-graded sense) by primitive elements $\omega_1, \ldots, \omega_\ell$, where ℓ is the rank of $\mathfrak{g}$. N. Hitchin conjectured that for any primitive element $\omega \in \wedge^d(\mathfrak{g}^*)^{\mathfrak{g}}$, there exists an irreducible $\mathfrak{sl}_2$-submodule V_ω of dimension d such that ω is non-zero on $\wedge^d(V_\omega)$.

We prove the validity of this conjecture for simple simply-laced Lie algebras implies its validity for any simple Lie algebra. Let G be a connected, simply-connected simple simply-laced algebrac group and K the fixed subgroup of a diagram automorphism of G. We show that the restriction map of representation rings, $R(G) \to R(K)$, is surjective. Our proof of the reduction of Hitchin's conjecture depends on this surjectivity.

Speaker: Kei Yuen Chan
Affiliation: University of Utah

Title: *Dirac cohomology for degenerate affine Hecke-Clifford algebras*

Abstract: The Dirac cohomology theory for graded affine Hecke algebras were introduced by Barbasch, Ciubotaru and Trapa. I will introduce a projective analogue of the Dirac operator for the degenerate affine Hecke-Clifford algebra. We shall also look at the action of the Dirac operator on some interesting degenerate affine Hecke-Clifford algebra modules in connection with the Dirac cohomology and Sergeev algebra modules.

Speaker: Christopher Drupieski
Affiliation: DePaul University

Title: *Polynomial superfunctors and universal extension classes for algebraic supergroups*

Abstract: In 1997, Friedlander and Suslin introduced the concept of strict polynomial functors, and then exhibited certain universal extension classes for the general linear group by way of explicit cohomology calculations in the category of strict polynomial functors. These extension classes were then used to prove that the cohomology ring of a finite group scheme is necessarily a finitely-generated algebra, and subsequently by other authors to establish various support variety results for arbitrary finite group schemes. In this talk I will discuss how the theory of strict polynomial superfunctors can be applied to exhibit universal extension classes for the general linear supergroup. Some of these 'super' universal extension classes restrict to the classes previously exhibited by Friedlander and Suslin, while others have no classical analogues.

Speaker: Houssein El Turkey
Affiliation: University of Oklahoma

Title: *Complexity of Modules over Lie Superalgebras*

Abstract: The complexity of a module is the rate of growth of a minimal projective resolution of the module. In this talk the complexity is computed for the Kac and the simple modules over the Lie superalgebra of type C. Then, a geometric interpretation is given via the theory of support varieties. We also compute the complexity of the simple modules over the exceptional Lie superalgebras D(2,1;α), G(3), and F(4).

Speaker: John Hutchens
Affiliation: Southern Arkansas University

Title: *k-involutions of algebraic groups of type F_4*

Abstract: We continue the classification of isomorphism classes of k-involutions of exceptional algebraic groups. In this paper we classify k-involutions for split groups of type F_4 over certain fields, and their fixed point groups. This classification of k-involutions is equivalent to the classification of symmetric k-varieties.

Speaker: Chun-Ju Lai
Affiliation: University of Virginia

Title: *Quantum Schur-type dualities of affine type B*

The Jimbo-Schur duality is a quantum generalization of the classical Schur duality between general linear Lie algebra and symmetric group. The deformed objects are quantum group/q-Schur algebra and Hecke algebra, respectively. In this talk, we will review the above-mentioned Schur-type dualities of finite type A, B and affine type A, and then talk about our work on the Schur-type dualities of affine type B. This is a joint work with L. Li and W. Wang.

Speaker: Chad Mangum
Affiliation: North Carolina State University

Title: *Representations of Twisted Toroidal Lie Algebras*

Abstract: Representations of Lie algebras, and particularly Kac-Moody algebras, have been significant in physics and other areas of mathematics for several decades. 2-Toroidal algebras are a generalization of affine Kac-Moody algebras which possess a realization as the central extension of a loop algebra (similar to the affine case) and can be viewed as the Lie algebra of polynomial maps from the torus into a simple, finite dimensional Lie algebra. This talk will discuss a different presentation of a subclass of toroidal algebras, as well as an interesting representation of that subclass. Sufficient introductory and background material will be included.

Speaker: Jacob Matherne
Affiliation: Lousiana State University

Title: *Computing Upper Cluster Algebras*

Abstract: Cluster algebras are commutative unital domains generated by distinguished elements called cluster variables. These generators are grouped into sets called clusters, and a process called mutation allows movement between the clusters. Many notable varieties (Grassmannians, partial flag varieties, and others) are equipped with cluster structures where certain regular functions play the role of cluster variables.

From a geometric perspective, there is a more natural algebra to consider: the upper cluster algebra. In this talk, we study cluster algebras and upper cluster algebras using algebraic geometry, which leads to an algorithm for producing presentations of upper cluster algebras in terms of generators and relations. This is joint work with Greg Muller.

Speaker: Elena Poletaeva
Affiliation: Univ. of Texas-Pan American

Title: *On Kostant's Theorem for the Lie Superalgebra $Q(n)$*

Abstract: A finite W-algebra is a certain associative algebra attached to a pair $(\mathfrak{g}, e)$, where $\mathfrak{g}$ is a complex semisimple Lie algebra and $e \in \mathfrak{g}$ is a nilpotent element. It is a result of B. Kostant that for a regular

nilpotent element e, the finite W-algebra coincides with the center of $U(\mathfrak{g})$.

We study finite W-algebras for basic classical superalgebras and $Q(n)$ associated to the regular even nilpotent coadjoint orbits. Kostant's theorem does not hold in this case. We show that this finite W-algebra satisfies the Amitsur-Levitzki identity and therefore all its irreducible representations are finite-dimensional. In the case of $Q(n)$ we give an explicit description of the finite W-algebra in terms of generators and relations and realize it as a quotient of the super-Yangian of $Q(1)$. This is a joint work with V. Serganova.

Speaker: Shifra Reif
Affiliation: University of Michigan

Title: *Kac Wakimoto Character Formula for the General Linear Superalgebra*

Abstract: In this talk, we shall discuss a new type of diagrams that enables one to track different highest weights of a module, their atypicality and corresponding characters. We use these diagrams to prove the Kac Wakimoto character formula, stated in 1994, for the general linear superalgebra and show that the modules to which it applies are exactly those for which all Kazhdan Lusztig polynomials are monomials. Joint with M. Chmutov and C. Hoyt.

Speaker: Amber Russell
Affiliation: University of Georgia

Title: *Lusztig's Generalized Green Functions*

Abstract: For a reductive algebraic group in good characteristic, Lusztig's generalized Springer Correspondence and his work proving the cleanness of character sheaves leads to an orthogonal decomposition of the equivariant derived category of constructible sheaves on the nilpotent cone. This orthogonal decomposition can be used to tell us about which local systems (on each orbit) appear as restrictions of the simple perverse sheaves. This information is encoded in Lusztig's generalized Green functions, and in this talk, we will make evident this fact.

Speaker: David Stewart
Affiliation: Cambridge University

Title: *On the Smoothness of Normalisers*

Abstract: Let G be an affine group scheme over a field k. We say G is smooth if $\dim \mathrm{Lie}(G) = \dim G$. A famous theorem of Cartier states that every affine group over a field of characteristic zero is smooth. In particular, the category of smooth group schemes is closed under the usual group theoretic constructions of taking centres, centralisers, normalisers and transporters. However, Cartier's theorem fails rather comprehensively in positive characteristic. A classic example of a non-smooth algebraic group is the group scheme μ_p whose points are the pth roots of unity; this is not smooth over a field of characteristic p.

Nonetheless, the centralisers of subgroup schemes of a reductive group G are usually smooth: work of Herpel gives precise information on the characteristic p of k, depending on the root datum of G, for all subgroup schemes of G to be smooth. It suffices, for instance, for p to be very good for G. In particular, all centralisers of subgroup schemes of GL_n are smooth.

The situation for normalisers is much less straightforward. For example, even when $G = GL_n$, for any $n \geq 3$ and any $p > 0$ an arbitrary prime, there are connected smooth subgroups of G with non-smooth normalisers. In light of this situation, perhaps it is surprising that there are any general situations in which normalisers of subgroup schemes are smooth. However, we prove that for sufficiently generic p depending on the root system of a reductive algebraic group G, all normalisers of height one subgroup schemes (in fact the normalisers of all subalgebras of the Lie algebra of G) are smooth and all normalisers of reductive subgroups are also smooth.

This theorem is proved by cohomological methods building on work of Bendel–Nakano–Pillen and Jantzen and Serre. There are consequences for the subalgebra structure of modular Lie algebras. This is joint work with Sebastian Herpel.

Speaker: Andrew Talian
Affiliation: University of Georgia

Title: *Endotrivial modules for classical Lie superalgebras*

Abstract: A supermodule M for a Lie superalgebra $\mathfrak{g}$ (over a field k) is called endotrivial if $\mathrm{End}_k(M) \cong k \oplus P$ for some projective supermodule

P. The set of such modules for a Lie superalgebra $\mathfrak{g}$ form a group under the tensor product, denoted $T(\mathfrak{g})$. In this talk, we classify the group of endotrivial modules for certain Lie superalgebras of particular interest called detecting subalgebras, and discuss progress towards classifying the group $T(\mathfrak{gl}(n|n))$.