SE Lie Theory Workshop 2014: Plenary Talks

Speaker: Shun-Jen Cheng
Affiliation: Academia Sinica, Taiwan

Title: *Irreducible Characters of Kac-Moody Lie superalgebras*

Abstract: Generalizing the super duality formalism for finite-dimensional Lie superalgebras of type $ABCD$, we establish an equivalence between parabolic BGG categories of a Kac-Moody Lie superalgebra and a Kac-Moody Lie algebra. The characters and Kostant $\mathfrak{u}$-homology groups for a large family of irreducible highest weight modules over a symmetrizable Kac-Moody Lie superalgebra are then given in terms of Kazhdan-Lusztig polynomials. This is a joint work with Jae-Hoon Kwon and Weiqiang Wang.

Speaker: Dimitar Grantcharov
Affiliation: University of Texas, Arlington

Title: *Weight modules of algebras of twisted differential operators on the projective space*

Abstract: We study blocks of categories of weight and generalized weight modules of algebras of twisted differential operators on $\mathbb{P}^n$. Necessary and sufficient conditions for these blocks to be tame and to be Koszul are provided. We also establish equivalences of categories between these blocks and categories of bounded and generalized bounded weight $\mathfrak{sl}(n+1)$-modules in the cases of nonintegral and singular central character. This is a joint work with V. Serganova.

Speaker: Jonathan Kujawa
Affiliation: University of Oklahoma

Title: *The hidden geometries of classical Lie superalgebras*

Abstract: Geometry provides a powerful tool in representation theory. In the past several decades the introduction of geometric tools was an important step forward for both finite groups and Lie algebras. Motivated by this success, Boe, Nakano and I and, independently, Duflo-Serganova, developed analogues for the classical Lie superalgebras. I will give an overview of the constructions, results, and conjectures in

this area. In particular, unlike in the previously studied settings, there seems to be several different geometries in play. The precise relationship between them remains a mystery.

Speaker: Shrawan Kumar
Affiliation: University of North Carolina

Title: *The multiplicative eigenvalue problem and deformed quantum cohomology*

Abstract: This is a joint work with Prakash Belkale. Let G be a simple, connected, simply-connected complex algebraic group. Choose a Borel subgroup B and a maximal torus $H \subset B$. We denote the Lie algebras of G, B, H by the corresponding Gothic characters: $\mathfrak{g}, \mathfrak{b}, \mathfrak{h}$ respectively. Let $R \subset \mathfrak{h}^*$ be the set of roots of $\mathfrak{g}$ and let R^+ be the set of positive roots (i.e., the set of roots of $\mathfrak{b}$). We denote by Δ the set of simple roots $\{\alpha_1, \ldots, \alpha_\ell\}$.

Consider the *fundamental alcove* $\mathcal{A} \subset \mathfrak{h}$ defined by

$$\mathcal{A} = \{\mu \in \mathfrak{h} : \alpha_i(\mu) \geq 0 \text{ and } \theta(\mu) \leq 1\},$$

where θ is the highest root of $\mathfrak{g}$. Then, $\mathcal{A}$ parameterizes the K-conjugacy classes of K under the map $C : \mathcal{A} \to K/\mathrm{Ad}\ K$,

$$\mu \mapsto c(Exp(2\pi i\mu)),$$

where K is a maximal compact subgroup of G and $c(Exp(2\pi i\mu))$ denotes the K-conjugacy class of $Exp(2\pi i\mu)$. Fix a positive integer $n \geq 2$ and define the *multiplicative polytope*

$$\mathcal{C}_n := \{(\mu_1, \ldots, \mu_n) \in \mathcal{A}^n : 1 \in C(\mu_1) \ldots C(\mu_n)\}.$$

Then, $\mathcal{C}_n$ is a rational convex polytope with nonempty interior in $\mathfrak{h}^n$. Our main result describes the facets (i.e., the codimension one faces) of $\mathcal{C}_n$. We construct deformations of the small quantum cohomology rings of homogeneous spaces G/P, and obtain an irredundant set of inequalities determining $\mathcal{C}_n$. The result was proved by Biswas in the case $G = SL_2$; by Belkale for $G = SL_m$ (and in this case a slightly weaker result by Agnihotri-Woodward); and by Teleman-Woodward for general G (though the result of Teleman-Woodward produced a set of inequalities which has redundancies in general).

Speaker: Cornelius Pillen
Affiliation: University of South Alabama

Title: *Vanishing cohomology and a new look at rational stability and generic cohomology*

Abstract: Let G be a connected reductive algebraic group over an algebraically closed field of positive characteristic and B be a Borel subgroup. In this talk certain vanishing results for B-cohomology are presented. As an application one obtains a new proof of the seminal work of Cline, Parshall, Scott and van der Kallen on rational stability and generic cohomology. Furthermore, vanishing ranges for the cohomology of the associated finite groups of Lie type are established which strengthen earlier results by Hiller.

Speaker: Laura Rider
Affiliation: M.I.T.

Title: *Modular representation theory and the geometric Satake equivalence*

Abstract: The geometric Satake equivalence proven by Mirkovic–Vilonen (building on work of Lusztig and Ginzburg) allows one to study the representation theory of an algebraic group via the topology of an infinite dimensional variety, known as the affine Grassmannian for the Langlands dual group. It is hoped that the modular representation theory of an algebraic group may be understood geometrically, with a key role played by a class of objects known as parity sheaves, defined by Juteau–Mautner–Williamson. In my talk, I will describe this dictionary between representation theory and geometry, with an emphasis on the Mirkovic–Vilonen conjecture, recently proven in joint work with Pramod Achar.

Speaker: Leonard Scott
Affiliation: University of Virginia

Title: *Forced gradings and Q-Koszul algebras*

Abstract: This talk is about recent work with Brian Parshall. In particular, I will highlight three new conjectures on the structure of finite-dimensional algebras A arising in the study of algebraic groups in positive characteristic, sometimes called "generalized Schur algebras."

Actually, the focus is not on the algebras themselves, but on "forced graded" versions $\widetilde{\mathrm{gr}}\, A$ of them, a construction here requiring filtrations of A arising from quantum groups. However, part of the conjectures is that these algebras $\widetilde{\mathrm{gr}}\, A$ share many properties with the original algebras A, including quasi-heredity and many homological invariants. One of the conjectures is that an important family of these invariants can be calculated by Kazhdan-Lusztig polynomials, even though fairly weak restrictions are required on the characteristic p (none at all in type A). It is also conjectured that these algebras $\widetilde{\mathrm{gr}}\, A$, with the same mild restrictions on p, are "Q-Koszul." This is a generalization we invented of the Koszul property to describe positively graded algebras which resemble Koszul algebras, but are quasi-hereditary, rather than semisimple, in grade 0.

These conjectures have an interesting history and relationship to other results of myself and Parshall, some of them related to the Lie algebra theme of this conference. I will also discuss these as time permits.

Speaker: Weiqiang Wang
Affiliation: University of Virginia

Title: *Super Kazhdan-Lusztig theory ABC*

Abstract: In the two talks, we describe Schur-Jimbo duality and its generalization, which is a duality between a coideal subalgebra of the quantum group of type A and the Hecke algebra of type B. We will discuss Brundan's Kazhdan-Lusztig conjecture for gl(m—n) via the canonical basis and its proof via super duality. The type B duality leads a new approach for Kazhdan-Lusztig theory of type B, via a new theory of canonical basis arising from quantum symmetric pairs which we develop. As an application, we formulate and establish a Kazhdan-Lusztig theory for the ortho-symplectic Lie superalgebras for the first time. The type B part is joint work with Huanchen Bao (PhD student at UVA), and the type A part is joint work with Shuni Cheng and Ngau Lam.